On intermittent fluctuations of the earth's fluid core motions

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Abstract: On the basis of the analysis of a 100 year record of monthly means of the geomagnetic field vertical component measured at the Niemegk observatory, the nature of MHD turbulence within the earth's fluid metallic core is investigated. It is shown that, for a subset of singularity exponents, the energy transfer rate between the scales is intermittent and non-homogeneous, this being in agreement with the predictions of the simple multifractal p-model.

Key words: fluid core turbulence, coherent convective motions, multifractal spectrum, scaling properties

1. Introduction

One of the main goals of geomagnetism is to determine the state of the earth's fluid core investigating the observed features of field variations at the earth's surface. Such observations have revealed that the earth's magnetic field continuously undergoes long-term secular changes in magnitude and direction, which could not be interpreted in terms of permanent magnetization of rocks. As a matter of fact, the magnetic field is stated to be generated by nonlinear interactions between the magnetic field and fluid motion within the earth's electrically conducting core. This is the basic concept formulated and thoroughly developed in dynamo theories *(Rikitake*

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and *Honkura, 1985)*. Usually, it is assumed that a turbulent or convective fluid motion is present in earth's liquid core allowing dynamo actions.

As early as 1948 Bullard has suggested that main features of the world-wide secular variations can be produced by a distribution of dipoles corresponding to turbulent eddies starting up and disappearing over the surface of the core *(Bullard, 1948)*. Investigating large magnetic Reynolds number dynamos, *Gubbins (1973)* has pointed out the importance of the interactions between small scale motions and large-scale magnetic fields. In his early paper Gubbins considered only a two-scale mechanism because of limited computer capability. Recent nonlinear theories on hydrodynamic and MHD turbulence developed for the description of laboratory and space fluid systems, however, treat multi-scale interactions describing canonical cascades of energy and other non-linearly conserved fluxes from the outer (macroscopic) scale into smaller and smaller scales generating non-homogeneous distributions of vortices (eddies) down to the dissipative scale *(Biskamp, 1993; Frisch, 1995)*. In accordance with this *De Michelis et al. (1998)* analysed the occurrence of sign singularity in the geomagnetic field secular acceleration (the second time derivative of the geomagnetic field). They have investigated the scaling properties of the so-called cancellation index which characterizes the sign singularity of a signed measure useful for describing random fields oscillating in sign. Their results give a clear evidence of the turbulent nature and intermittent character of the velocity fluctuations of the fluid core motion.

In order to better understand the intermittent character of the energy transfer between scales within the earth's fluid core, we introduce and analyse a positive probability measure in this paper. To this end we use the so-called large deviation multifractal spectrum technique introduced recently by *Véhel and Vojak (1998)* and by *Canus et al. (1998)*. Then a multifractal model is shown to be fitted to experimental results.

2. Large deviation multifractal spectrum

In the next paragraph we will introduce a probability measure μ defined on the unit interval $[0,1)$ using a time series from the Niemegk geomagnetic observatory. In order to introduce the continuous large deviation multifractal spectrum (LDMS) *(Véhel* and *Vojak, 1998; Canus et al., 1998)* we consider $P := (P_n)_{n\geq 1}$; the sequence of partitions P_n of the unit interval $[0, 1)$. P_n is constructed as

$$P_n := (I_n^k) \qquad \text{and} \qquad 0 \leq k \leq 2^n - 1 \tag{1}$$

with

$$I_n^k := [k\ 2^{-n}, (k+1)\ 2^{-n}). \tag{2}$$

As a following step we use the concepts of Hölder exponents and singularity spectra. The Hölder exponent (or singularity exponent, also crowding index) is defined at a point $x_0 \in \mathrm{Supp}(\mu)$ as a limit *(Muzy et al., 1994)*:

$$\alpha(x_0) = \lim_{\eta \to 0} \frac{\log \mu(B_{x_0}(\eta))}{\log \eta}, \tag{3}$$

where $B_{x_0}(\eta)$ is the ball centered at x_0, the size being η, and $\mathrm{Supp}(\mu)$ denotes the support of the measure μ. If μ is defined e.g. as a probability measure, Eq. (3) expresses the power law dependence of probability measure on resolution η with a set of Hölder exponents $\alpha(x_0)$ *(Tél, 1988)*.

The so-called $f(\alpha)$ singularity spectrum of a measure μ associates to any given α, the Haudorff dimension of the set of all the points x_0, which are such that $\alpha(x_0) = \alpha$:

$$f(\alpha) := d_H(x_0 \in \mathrm{Supp}(\mu), \alpha(x_0) = \alpha) \tag{4}$$

with the number of subsets N_α with the same α *(Tél, 1988)*:

$$N_\alpha(\eta) \sim \eta^{-f(\alpha)}. \tag{5}$$

Now, for every interval I of size $|I| = \eta$ let $\alpha_\eta(I) := \log \mu(I)/\log \eta$ be the coarse grain Hölder exponent of I. Then a Lebesgue measure is computed being the reunion of all intervals of the same size for which the coarse grain Hölder exponent is equal to a Hölder exponent α. The Lebesgue measure is defined as *(Canus et al., 1998)*:

$$p_\eta^c(\alpha) := |E_\eta(\alpha)|, \tag{6}$$

where the set E_η is

$$E_\eta(\alpha) := \bigcup \{I \in [0, 1) : |I| = \eta, \alpha_\eta(I) = \alpha\}. \tag{7}$$

It is possible to introduce an ε precision for coarse grain Hölder exponent by

$$\varepsilon \geq |\alpha_\eta(I) - \alpha| \tag{8}$$

and compute the Lebesgue measure as

$$p_\eta^{c,\varepsilon} := \int_{\alpha-\varepsilon}^{\alpha+\varepsilon} p_\eta^c(\beta)\mathrm{d}\beta, \tag{9}$$

for which the continuous (superscript c) LDMS is *(Canus et al., 1998)*

$$f_g^c(\alpha) := 1 - \lim_{\varepsilon \to 0} \lim_{\eta \to 0} \frac{\log p_\eta^{c,\varepsilon}(\alpha)}{\log \eta}. \tag{10}$$

In this way LDMS is closely related to the Hausdorff spectrum of dimensions describing interwoven fractal subsets with singularities α and dimensions $f(\alpha)$.

3. Data analysis

The definition of a physical quantity which may be distributed on a given support depends on the physical system under consideration. In the studies of the fully developed turbulence, usually the velocity difference inside eddies is a quantity defining a measure *(Tél, 1988; Biskamp, 1993; Frisch, 1995)* or a signed measure *(De Michelis et al., 1998)*. Usually, the *q-th* order structure functions $S^q(\eta) \sim \langle |v(x+\eta) - v(x)|^q \rangle$ are computed. Then from the scaling relation of $S_q(\eta) \sim \eta^{\xi}(q)$ information on intermittent non-homogeneous energy transfer rate between nonlinearly interacting eddies of a different size can be obtained if the exponent function $\xi(q)$ is nonlinear *(Paladin* and *Vulpiani, 1987)*.

In this paper we estimate the turbulent energy transfer rate considering a turbulent eddy of a temporal size (time delay) τ

$$\Phi_B(t_i) \sim |B(t_i + \tau) - B(t_i)|^2. \tag{11}$$

Assuming that Taylor's frozen field hypothesis is valid, the temporal variations of geomagnetic secular changes should reflect the spatial one, though, the field variations are measured outside of the region (at the surface) where the field itself is generated by dynamo action (in a liquid core). The measure introduced through magnetic field differences rather than velocity fluctuations approximates the expression for the velocity structure function quite well in the case of MHD turbulence *(Meneveau* and *Sreenivasan, 1987)*. In

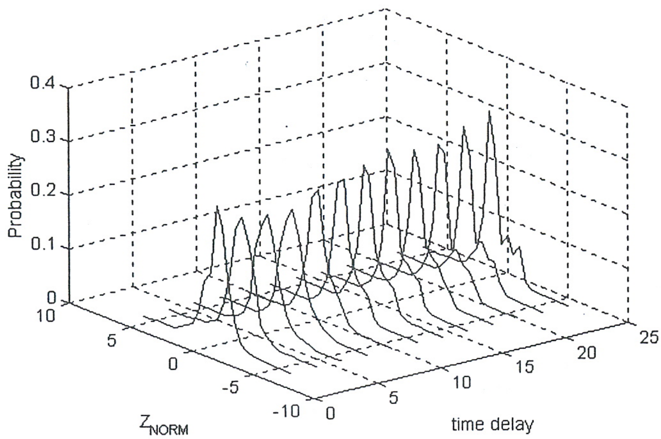

Fig. 1. Empirical probability distribution functions for various time delays $\tau = 2, 4, 6, ...24$ months.

fact, the nonlinear interaction between the magnetic field (B) and fluid motion (v) is the basic working assumption in the dynamo theory. Therefore, the introduced positive measure of the square of magnetic field differences might allow an analysis of intermittency effects in the liquid core.

We use 100 year long (from 1897) monthly means based on registration of the Z-component of the geomagnetic field at the Niemegk observatory. In case of utilization of monthly means of the magnetic field vertical component, the contamination due to the external field sources is substantially reduced. Low-pass filtering removes the remaining external influence introduced by solar cycle variations. After that the time series was normalized.

Fig. 1 shows the empirical probability distribution function (EPDF) computed for different values of the time delay $\tau = 2, 4, 6, ..., 24$ months. For eddies of smaller time span (low values of τ) EPDF is quasi-Gaussian. For higher values of τ, however, a long tailed EPDF seems to be stable. An inspection of EPDFs for larger values of τ is limited by the size of the available data set. Because of the existence of these two different regimes, LDMS is computed for the sets of values $\tau \in [2, 12]$ and $\tau \in [14, 24]$ months, respectively.

A probability measure $\mu(i) \equiv \mu(\Phi_Z(t_i))$ on the unit interval $[0, 1)$ is constructed, which corresponds to the realization of an average dissipation rate at the moment t_i:

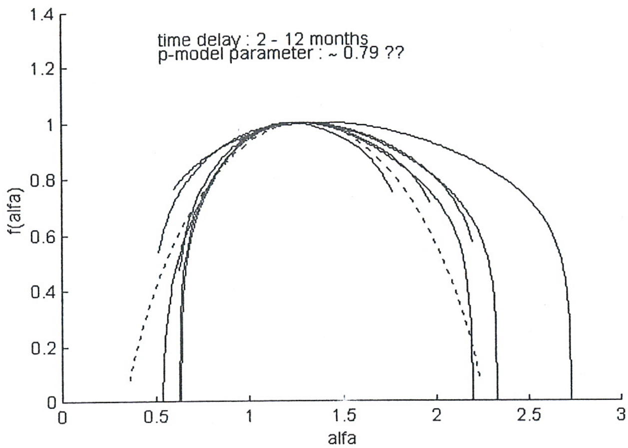

Fig. 2. Large deviation multifractal spectra for $\tau \in [2, 12]$ months. The p-model fit is rather poor.

$$\mu(i) = \frac{\Phi_Z(t_i)}{\sum_i \Phi_Z(t_i)} \tag{12}$$

and

$$\sum_i \mu(i) = 1. \tag{13}$$

Continuous LDMS is estimated using relation (10).

Fig. 2 shows the computed $f(\alpha)$ spectra for the values of $\tau \in [2, 12]$ months. A significant scattering of the curves is evident and fitting a theoretical model (dotted line) to the observations is not straightforward.

For larger values of $\tau \in [14, 24]$ months, the so-called p-model was fitted to the observations giving the best fit for singularities of $\alpha \in [0.85, 1.8]$ (Fig. 3). The better estimations of $f(\alpha)$ require larger data sets. The smaller the exponent α, the more singular the measure and the stronger the singularity. The limit $\alpha = 0$ corresponds to the singularity of the Dirac distribution.

The p-model was introduced by *Meneveau* and *Sreenivasan (1987)* as a simple multifractal model for the description of energy cascade processes in fully developed turbulent flows. For the sake of simplicity we outline only the basic assumptions of this model. The largest turbulent eddy of size L is assumed to be built up by a specific energy flux per unit length. Then

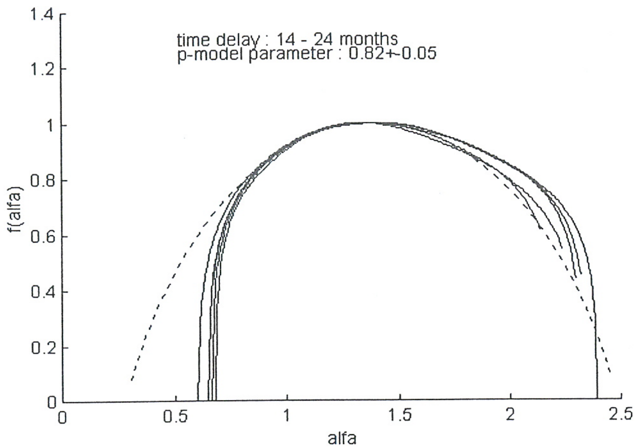

Fig. 3. Large deviation multifractal spectra for $\tau \in [14, 24]$ months. The p-model fit gives $p = 0.82 \pm 0.05$ corresponding to strong intermittency effects.

a scale independent space averaged cascade rate is considered and the flux density is transferred to the two smaller eddies with the same scale length, but different flux probabilities P_1 and P_2 ($P_1 + P_2 = 1$). This process with randomly distributed P_1 and P_2 is repeated again and again. Then the parameter $p = P_1 = 1 - P_2$ governs the asymmetric breakdown in the fragmentation process. An analytical expression for the p-model with equal scales $l_n = L/2^n$ but unequal weights is given by *Halsey et al. (1986)*

$$\alpha = -\frac{\log_2 p + (w-1)\log_2(1-p)}{w}, \tag{14}$$

$$f(\alpha) = -\frac{(w-1)\log_2(w-1) - w\log_2 w}{w}, \tag{15}$$

where w is a free parameter. The value of $p = P_1 = P_2 = 0.5$ corresponds to the homogeneous energy transfer rate with no intermittency effects. By increasing the value of p intermittency increases.

From Fig. 3 we recover parameter $p = 0.82 \pm 0.05$ which gives evidence of strong intermittency of the energy flux distribution within earth's fluid core, though, only for a subset of Hölder exponents. The fact that $f(\alpha)$ attains its maximum at $\alpha \sim 1.38 \pm 0.02$ means that the energy dissipation field is not space-filling, but singular. In other words, the inhomogeneous distribution of the measure $\mu(i)$ takes place on fractal support.

4. Conclusions

Our preliminary analysis, based on the estimation of multifractal characteristics (LDMS) of geomagnetic data, reveals important scaling laws present in magnetic field secular changes. The obtained $f(\alpha)$ spectrum gives evidence for multifractal probability measure distributed on fractal support. We believe that the underlying scaling laws are the consequence of a nonhomogeneous energy dissipation field within earth's fluid core leading to singularities on small scales.

Our results qualitatively confirm the findings on intermittency reported by *De Michelis et al. (1998)*. However, stronger intermittency effects are shown in this study. The p-model fit in their case gives the value of $p \sim 0.75$. It can be explained by the fact that we have introduced a different kind of positive measure. We are aware of the limitations introduced by the size of the analysed data set and we plan to extend our analysis by considering data from several geomagnetic observatories.

We intend to better understand the anisotropic nature of the fluid core turbulent motions and the effect of the differential rotation. A fundamental issue is to understand how the rotation affects turbulence within the earth core. Namely, in case of uniform rotation, the usual 3D isotropic turbulence can collapse to the 2D turbulence in which both energy and entropy are conserved by the nonlinear terms, hence both undergo to be cascaded *(Schertzer* and *Lovejoy, 1993)*. Numerical simulations have verified the existence of inverse energy cascades in 2D turbulence building-up self-organized coherent convective structures at the longest wavelength *(Hossain, 1994)*. An experimental verification of these questions will be the subject of future investigations.

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