Geomagnetic diagnosis of the Earth's fluid core turbulence on the basis of large deviations

Z. Vörös, M. Váczyová
Hurbanovo GO, Geophysical Institute of the Slovak Academy of Sciences[1]

J. C. Gianibelli, I. R. Cabassi
La Plata National University[2]

A b s t r a c t :The state of the Earth's fluid core is usually investigated on the basis of the observed features of geomagnetic variations at the Earth's surface. This task also involves difficulties associated with shorter time scale variations of internal origin which appear to be undistinguishable from the external fields, at least on the time scale of years or decades. In this paper a new method is proposed which may separate internal and external field variations on the basis of their scaling and singularity properties.

Key words: geomagnetic field, multifractal scaling, MHD turbulence

1. Introduction

The shortest variations of the internal field that can be observed at the Earth's surface thought to be determined by the electrical conductivity of the mantle. The primary conception about the spectrum of time variations has considered their sources in a way that on time scales shorter than the secular changes all variations are due to the fields of external origin (substorms, storms, solar cycle modulation, etc.), and on longer time scales variations are due to the internal changes. It has been found recently, however, that some kinds of core dynamics may operate much more quickly, on time scale of decades, e.g. torsional oscillations (*Zatman* and *Bloxham, 1999*), or on the scale of a year or less e.g. geomagnetic jerks (*Alexandrescu et al., 1995*). *Barraclough* and *De Santis (1997)* have also pointed out that the geomagnetic field evolves as a non-linear system with unpredictable behaviour after times greater than a few years. Consequently, the temporal behaviour of the geomagnetic main field is not well understood and there can exist -an

[1] 94701 Hurbanovo, Slovak Republic, e-mail: geomag@geomag.sk
[2]Faculty of Astronomical and Geophysical Sciences, Department of Geomagnetism, 1900 La Plata, Argentine, e-mail: jcg@fcaglp.fcaglp.unlp.edu.ar

overlapping frequency region within which the internal and external time variations detected at the Earth's surface appear to be mixed.

In this paper we pose the question if the contributions of the external and internal fields can be separated. In other words, we are looking for a proper method sensitive enough for detecting those characteristics of time variations which are different for the external and the internal fields and which are likely to remain hidden as far as spectral analysis is concerned. It is likely that recent results on MHD turbulence make such a separation possible. Let us consider the MHD energy transfer and dissipation processes in that manner. The matter in question is that the free energy may be distributed in hydrodynamic or MHD systems in intermittent and fluctuating manner leading to self-organization, nonlinearities and singularities, that is to say, to multi-scale phenomena which requires to perform a multi-scale analysis of the data (*Consolini et al., 1996; Biskamp,1997; Chapman et al., 1998; De Michelis et al., 1998; Vörös et al., 1998; Chang, 1999; Klimas et al., 2000; Vörös, 2000*). Below we use the so-called large deviation multifractal technique proposed by *Véhel* and *Vojak (1998)* and *Canus et al. (1998)*.

2. A multi-scale analysis method: large deviations

The adaptation of the method of the large deviation multifractal spectra to the multi-scale description of geomagnetic fluctuations is described in detail earlier *(Vörös, 2000)*. Here, we recapitulate only the main points. It is assumed that an amount of accumulated/dissipated geomagnetic signal energy during a time period T can be described by a probability measure:

$$\varepsilon(j) = \sum_{i}^{j=i+T} E(i).\Delta t \qquad \text{and} \qquad \sum_{j} \varepsilon(j) = 1, \tag{1}$$

where Δt is the time resolution and E(i) represents the normalized energy content of the geomagnetic signal given by ΔZ^2, previously differenced through:

$$\Delta Z = Z(t+\tau) - Z(t). \tag{2}$$

Eq. (2) represents a kind of filtering with a limiting frequency proportional to τ. It is worth mentioning that in turbulence studies the time structure of the velocity field measured in a point of a flow between instants separated by τ is usually related to the spatial structure of dissipation regions *(Frisch, 1996)*. The velocity difference inside dissipation structures (eddies) is a positive and additive quantity defining usually a singular measure with no density. More simply, the expression „singularity“ applies to describe nonhomogeneous, highly intermittent turbulent fields with dissipation regions containing an energy flux density $\phi_\lambda \rightarrow \infty$ at the scale λ . There is no energy dissipation within the regions where $\phi_\lambda \rightarrow 0$.

There exist several ways how to describe those intermittent turbulent fields with singular measures. Here we introduce the so-called Hölder exponents defined at a point $x_0 \in \mathrm{Supp}(\varepsilon)$ by

$$\alpha(x_0) = \lim_{\lambda \to 0} \{\log \varepsilon(B_{x_0}(\lambda)) / \log(\lambda) \}, \tag{3}$$

where $B_{x_0}(\lambda)$ is a ball centered at x_0 of size λ and $\mathrm{Supp}(\varepsilon)$ denotes the support of the measure ε (a set on which the measure is distributed). Eq. (3) expresses the power law dependence of probability measure on resolution λ with a set of Hölder exponents $\alpha(x_0)$. Another scaling law is related to the number of subsets N_α with the same α

$$N_\alpha(\lambda) \sim \lambda^{-f(\alpha)} . \tag{4}$$

Véhel and *Vojak (1998)* and *Canus et al. (1998)* proposed an algorithm for computing the large deviation singularity $(\alpha, f(\alpha))$ spectrum which relies on the computaion of the Lebesgue measure of the reunion of all intervals of same size for which the coarse grain Hölder exponent is equal to the Hölder exponent up to a δ precision. Without going into details we outline the basic features of large deviation spectra only. First of all, the singularity exponent α characterizes locally (at a point x_0) a time series, while $f(\alpha)$ describes the global distribution of interwoven subsets with singularity α. It is obvious from definition of α (Eq. 3) that the singular measure is of Dirac delta type for $\alpha(\lambda \rightarrow 0) = 0$. Also, the measure is more singular for $\alpha(x_0)<1$. Of key importance is the shape of $f(\alpha)$ as stated by *Vörös (2000)*. If it is a smooth concave function with a parabolic shape like the symbol $\cap$ then the singular measure is produced by a multi-scale energy cascade process with multiplicative rescaling structure. When the $f(\alpha)$ spectra are not exactly $\cap$-shaped deviations from the multiplicative model occur. It can be explained considering sums of multiplicative measures of disjoint supports in which case the large deviation singularity spectrum is simply the maximum of the individual spectra *(Riedi* and *Véhel, 1997)*.

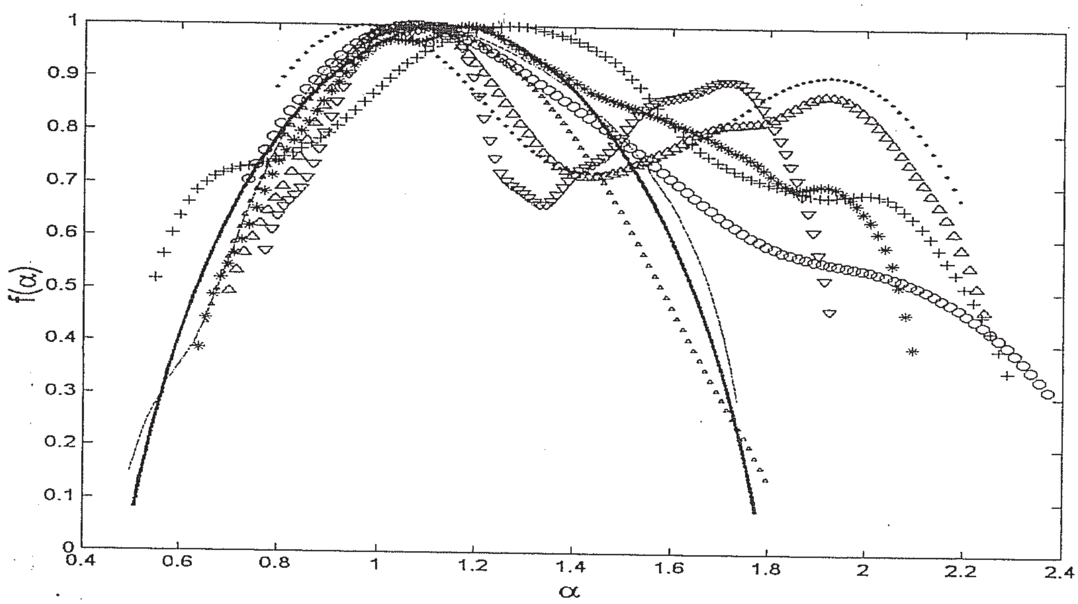

Fig. 1. Large deviation singularity spectra. The thick continuous line corresponds to the multiplicative P-model; the thin line displays the singularity spectrum on substorm/storm time scale (τ=60 min, T=300 min); Further profiles are: the singularity spectrum computed from Niemegk (smaller Δ, 1897-1998) Z component monthly mean data taking τ=5 months and T=5 months; the singularity spectra computed from Kakioka (+, 1897-1998), Chambon La Foret (*, 1883-1998), Niemegk (larger Δ, 1897-1998), Brorfelde (∇, 1907-1997), Las Acacias (..., 1961-1997), Hurbanovo (o, 1949-1999) Z component monthly mean data taking τ=36 months and T=5 months.

Vörös (2000) has shown that after transforming geomagnetic data using Eqs. (1) and (2) the singularity f(α) spectra for loading-unloading processes on the time scale of substorms and storms are of parabolic shape. Moreover, energy transfer processes from large to small scales can be described by the so-called multiplicative multi-scale P-model *(Meneveau* and *Sreenivasan, 1987).* The singularity spectra corresponding to the fluctuations of solar wind origin are a bit different. Now, we expect that if we choose properly the parameters τ and T, larger deviations from the singularity spectra of the external field fluctuations occur and these new properties correspond to the singularity characteristics of the internal field. In Fig. 1 the results using the data from six geomagnetic observatories are shown. It has to be stressed that there is no apriori knowledge about the shape of the singularity spectra corresponding to the variations of the internal field being represented by transformed data (Eqs. 1,2). Though the f(α) spectra for Niemegk observatory data have already been estimated by *Vörös* and *Gianibelli (1998),* the large deviations from a parabolic $\cap$-shaped multiplicative model were not considered. Moreover, Eq.(1) down-samples the data and some precaution is needed for achieving a more or less robust estimation of $f(\alpha)$ spectra. Therefore, we tested some combinations of the parameters T and τ for which the $f(\alpha)$ spectra for the data considered were at least similar. There were two limit cases found. The first one for small delay (e.g. τ=5 month, high-pass filtering) leads to the spectra which are very similar to the scaling laws exhibited by P-model (thick line) or which describe the singularity distribution for substorm/storm (external field) processes (thin line). This result is only depicted for Niemegk observatory data (smaller Δ symbols) as seen in Fig.1. The spectra for the other data are similar in case of small τ. For larger values of delay (e.g. τ=36 month in Fig.1) two peaks appear and we conjecture that the new distributions (with a maximum value somewhere between $1.5<\alpha<2.5$) are likely to correspond to the singularity/scaling characteristics of the internal field fluctuations. To support this idea the following arguments are essential: **a)** on the smaller time scales (smaller τ) the $f(\alpha)$ spectra are reduced to the $\cap$-shaped distribution known to be related to the multiplicative rescaling structure of the external field fluctuations; **b)** on the larger time scales (larger τ) the deviations from the simple $\cap$-shaped distribution are significant, but the external field spectra do not disappear; **c)** though the data sets of geomagnetic Z components by itself are rather short for a robust computation of large deviation spectra, all the estimates lead to the same qualitative result; **d)** the applied technique of large deviations allows to identify sums of multiplicative measures of disjoint supports; since different physical processes can generate different measures (distributions), possible models with similar characteristics as the observed spectra are likely to contain physical information on the contributing external and internal sources.

4. Conclusions

The large deviation multifractal analysis of singularity spectra of geomagnetic fluctuations were presented in detail. As a result different singularity distributions were obtained for internal and external field fluctuations. Previous results (*De Michelis et al., 1998; Vörös* and *Gianibelli, 1998)* gave an evidence on turbulent and intermittent character of velocity fluctuations of the fluid core motion leading to the classical conception of fluid turbulence in which energy cascades from large to small eddies similarly to the 1D P-model. Other authors, e.g. *Braginsky* and *Meytlis (1990)* precluded the possibility of cascade models for fluid core motion. Our experimental results on singularity distributions show that only a qualitative conclusion can be drawn regarding the separation of external and internal fields. A description of singularity distribution generated by Earth's fluid core motion in terms of multiplicative cascade models is not fully possible due to the limited data length. However, the knowledge of differencies in singularity distributions seems to be useful for MHD modeling of internal and external fields.

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